**Problem 1:**

For multi-stage seawater reverse osmosis (SWRO) with inter-stage booster pumps in which retentate pressure drop may be ignored,

1. Neglecting the effect of concentration polarization, prove that the lowest normalized specific energy consumption (NSEC) may be described by

where is the dimensionless pressure at the entrance of stage j, is the fractional recovery in stage j, and is the efficiency of the energy recovery device (ERD).

1. Prove that as N→ ∞, → ∞ and → 100%, the NSEC in a continuous multi-stage SWRO with inter-stage booster pumps and ERD is equivalent to the one in a fully reversible RO process, i.e.
2. Explain why there is still a huge gap between SEC in industrial operation and the one suggested by theory.

**Solution to Problem 1:**

1. For a generic multi-stage SWRO shown in Figure 1, you may determine the process parameters in Table 1.



Fig. 1 Schematic of a multi-stage SWRO with inter-stage booster pumps.

Table 1. Expression of parameters in single or multi-stage RO processes.

|  |  |  |
| --- | --- | --- |
| Stage | 1 |  |
| (for pump) |  |  |
| (for RO) |  |  |
| (recovery) |  |  |
| (membrane area) |  |  |
| (transmembrane osmotic pressure at inlet) |  |  |
| (feed rate) |  |  |
| (permeate rate) |  |  |
| (dimensionless pressure) |  |  |
| (membrane capacity production ratio) |  |  |
| (membrane capacity intake ratio) |  |  |
| (overall recovery) |  |  |
| (work recovered by ERD) |  |  |
|  |  |  |

Therefore, NSEC ignoring pump efficiency is:

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

where is transmembrane osmotic/hydraulic pressure ratio at stage (i.e. ), and is fractional recovery at stage . Equation (1) was derived based on the total pump energy consumption divided by the total permeate production and the transmembrane osmotic pressure at the feed location. It is valid even if the number of RO stages is one or no ERD is used.

1. At the thermodynamic limit (where is sufficiently large), for . The NSEC may be simplified as follows:

|  |  |  |
| --- | --- | --- |
|  |  | (2) |

The minimization of NSEC is subject to a specified overall recovery ():

|  |  |  |
| --- | --- | --- |
|  |  | (3) |

The optimal solution is determined by taking the partial derivative of NSEC with respect to () and solving all the resulting equations. It is shown that:

|  |  |  |
| --- | --- | --- |
|  |  | (4) |

Therefore, the minimum of NSEC at the thermodynamic limit is:

|  |  |  |
| --- | --- | --- |
|  |  | (5) |

For infinite number of RO stages,

|  |  |  |
| --- | --- | --- |
|  |  | (6) |

When ,

|  |  |  |
| --- | --- | --- |
|  |  | (7) |

This is equivalent to the NSEC in a fully reversible RO (shown in Figure 2)

|  |  |  |
| --- | --- | --- |
|  |  | (8) |



Fig. 2 A fully thermodynamically reversible RO where at any time.

1. Based on the above analysis, to reach the theoretical limit implied by Eq. (8), the following requirements should be met in the continuous operation of RO:
2. Infinite number of stages
3. Infinite in each stage
4. No pressure drop in retentate stream
5. 100% ERD
6. 100% pump efficiency
7. No concentration polarization

While the above are not feasible in a practical design, it is definitely possible to reduce energy consumption by:

1. Employing ERD, especially when overall water recovery is low.
2. Using two- or three-stage design with inter-stage booster pumps, especially when overall water recovery is high. It is not necessary to use five or more stages as the saving in energy is minimal.
3. Increasing at each stage (by increasing membrane area or utilizing more permeable membranes) so that the operation is closer to the thermodynamic limit. The does not have to be very large because the saving in energy consumption may be minimal when is sufficiently large.
4. Investigating dynamic operation techniques, e.g., closed-circuit desalination. But it is important to note that brine recirculation generally does not reduce energy consumption because of increase in feed salinity and retentate pressure drop.